A New Approach to Quasi-Static Analysis with Application to Microstrip

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Abstract—**The measured equation of invariance (MEI), a new, rapid technique for electromagnetic field analysis, is applied for the first time to planar circuit problems. Variations of the technique applicable to microstrip geometry are described. As a demonstration, it is then applied to electrostatic analysis of structures held at different potentials, using novel umbilical meshes. The MEI technique offers the possibility of order-of-magnitude increases in computational speed for typical problems of microwave CAD.**

I. Introduction

The measured equation of Invariance (MEI) technique is new approach to electromagnetic field calculations, invented by Mei *et al.* [1], and demonstrated in applications to scattering by conducting and dielectric bodies [2]. It is remarkable because it eliminates most of the mesh points needed in the conventional finite-difference technique, resulting in much faster computation. The reduction in mesh size is much greater than with absorbing boundary conditions, and in addition, MEI has a dramatic ability to include in the calculation conductors or boundary conditions which are completely outside of the mesh! There is also some similarity to the Method of Moments, in that both make use of an appropriate Green's function. However, the MEI method results in sparse matrices, yielding much faster solution times for large problems. The MEI technique has been found to give highly accurate results in a variety of problems (See, for example, Fig. 3.) It should be noted that what is being used here is not simply a slight modification of well known methods. As announced in reference [1], it is an entirely new technique for electromagnetic field problems.

The purpose of this letter is to demonstrate and present recent advances in the use of the new method for problems of the sort that rise in planar microstrip circuits. Although it is by no means necessary, we restrict ourselves for the moment to the quasi-static approximation, solving for the potential, Φ, satisfying Laplace's equation subject to Φ = constant on the microstrip, $\Phi = 0$ on the ground plane, and tangential electric field continuous across the dielectric interface.

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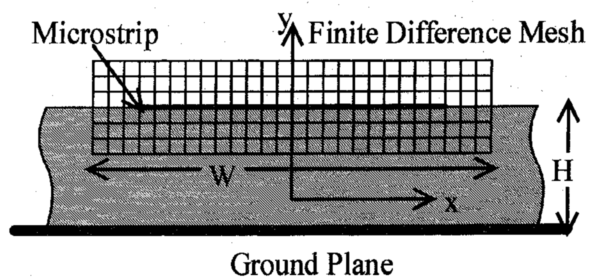

Fig. 1. Microstrip structure, with the entire finite-difference mesh used with the MEI mehtod. Note that the mesh covers only the microstrip line.

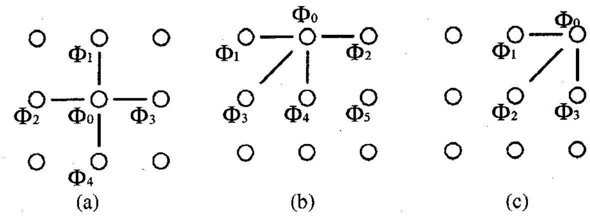

Fig. 2. Geometry of local nodes for writing finite-difference equations.

II. Outline of the Method

The MEI method uses information from the Green's function to terminate finite-difference meshes extremely closely to the objects being analyzed. The finite-difference mesh used for microstrip is shown in Fig. 1. Note that only the microstrip is enclosed by the mesh, while most of the dielectric interface and all of the ground plane are outside the mesh.

Finite-difference equations (FD) are written for interior points of the mesh. Thus, we use for the points shown in Fig. 2(a),

$$\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 - 4\Phi_0 = 0. \tag{1}$$

We may write this "equation of invariance" in the more general form,

$$\sum_{i=1}^{N} a_i \Phi_i = \Phi_0. \tag{2}$$

For points not on the dielectric interface, $N = 4$, and $a_i = 0.25$. For points along the dielectric interface, the coefficients are $a_1 = 1/2(1+\varepsilon_R), a_2 = a_3 = 0.25$, $a_4 = \varepsilon_R/2(1+\varepsilon_R)$.

On the mesh boundary, an equation for the set of points shown in Fig. 2(b) or (c) must be written. This equation must depend upon the external geometry of the problem. Otherwise, the answer would, for example, be independent of the height

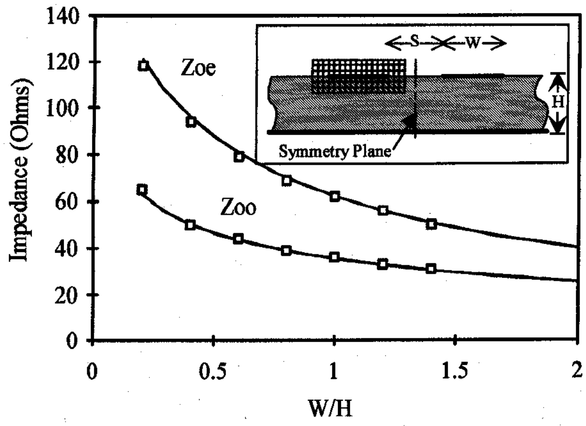

Fig. 3. Calculated even (Z_{oe}) and odd (Z_{oo}) mode characteristic impedances for coplanar coupled lines. The solid lines are this work, and the points are from [3]. For the case plotted, $S/H = 0.4$. Substrate permitivity is 9.6.

H, which clearly is not right. To obtain the coefficients, a_i, the Green's function that satisfies the external boundary conditions ($\Phi = 0$ on the ground plane and tangential E continuous on the dielectric surface, in this case) is used. Electric potentials, known as measuring functions, are constructed by integrating the Green's function against arbitrarily chosen charge distributions, which are called metrons. N sets of values Φ_i, corresponding to these constructed fields, are then inserted into (2), yielding a system of N simultaneous equations which can be solved for the coefficients $a_1, \cdots, a_N$. (N would be 5 for the case of Fig. 2(b), and 3 for the case of Fig. 2(c).)

Having found sets of coefficients, a_i, for each external mesh point (the a_i will be different for different mesh points), we can now find the actual potentials at the M lattice points. This is done by writing M simultaneous equations by generalizing the finite-difference method in the following way. Equation (2) is written for each mesh point. For internal points, the standard FD coefficients are used, while for external points, the measured equations are used with the appropriate coefficients, a_i, for each particular point. This yields a highly sparse system of equations, which may be more rapidly solved than an equivalent system using the method of moments, which yields full matrices.

III. The Metrons

The N measuring functions, $m_k(\vec{r})\{k = 1, \cdots, N\}$, are found by integrating a chosen metron, σ_k, against the Green's function for the external geometry, as

$$m_k(\vec{r}) = \int_C G(\vec{r}|\vec{r}')\sigma_k(\vec{r}')dr'. \tag{3}$$

The metrons are simply assumed charge distributions placed on the surface of the microstrip. Their purpose is to generate measuring functions satisfying the external boundary conditions. The Green's function in this case is the static Green's function using image theory from [3]. In most cases, the metrons we use are simple sinusoidal functions. The exact choice of metrons is not an important factor in the solution. Metrons that are continuous functions of position generally work well. It is interesting that the sets of coefficients, a_i, that emerge from the computation are nearly independent of the choice of metrons. The specific metrons used for each example will be given next.

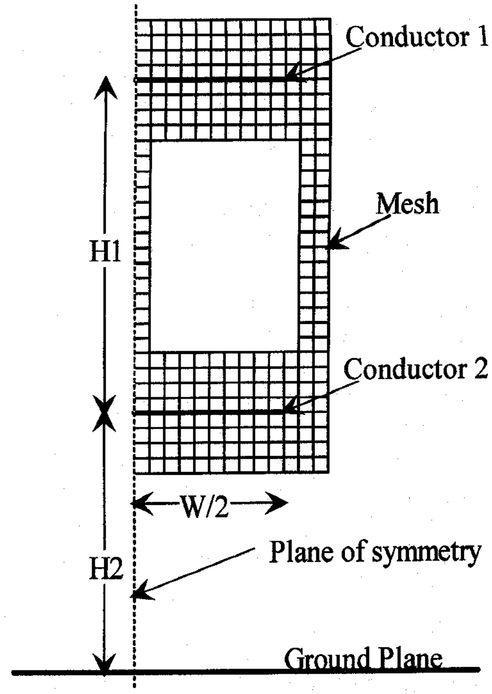

Fig. 4. Bilevel coupled lines. No dielectric is present in this case. For the dimensions $W/H_2 = 1.25$, $H_1/H_2 = 1.5$, the following results are obtained: $C_1 = 2.597$, $C_2 = 3.785$, and $C_m = 1.079$. Using the spectral domain approach [4], the results are 2.566, 3.753, and 1.065, respectively. Note that the entire finite-difference mesh used in the calculation is shown.

IV. Application to Microstrip

The potential is obtained by solving the sparse matrix yielded by the previous procedure. Once the potential is obtained, the electric field and charge distribution may be determined, and from this the impedance of the line calculated. Results for the impedance of a single microstrip are in extremely good agreement with known results [3], even with the coarse mesh shown in Fig. 1. The metrons used in this example were $\cos(2(i-1)\pi x/W)$, for $i = 1, 2, 3, 4, 5$. Using sin instead of cos, or multiplying by an edge condition factor, $1/\sqrt{x^2 - (W/2)^2}$), yields nearly identical results.

Completely different problems may be solved by changing the Green's function used to generate the measuring functions. Symmetric coupled microstrips, seen in the inset of Fig. 3, may be solved by using images to enforce even or odd symmetry, corresponding to the normal modes of the coupled lines. The results shown in Fig. 3 agree well with previous work [3]. This example illustrates the strength of the technique. Keeping the mesh, the FD equations, and the metrons the same, and changing only the Green's function to reflect the different outside dimensions, one generates a family of different curves.

V. Umbilical Meshes

More general coupled lines, where consideration of symmetry does not give the normal modes of the system, may be analyzed as well. In this case, both conductors must be surrounded by meshes, and two analyses performed in order to obtain the self and mutual capacitances of the conductors. First, the potential of one conductor is held at 1 volt and the other grounded, yielding C_1 and C_m, then the voltages are reversed, yielding C_2.

Two separate meshes are not quite sufficient to solve the problem. The potentials near each conductor are influenced by the voltage of the other. The measured equations do not contain any information about the potential of the conductors, so if the two meshes were completely distinct, the solution near one conductor would be independent of the voltage on the other. Hence, we connect the two meshes with "umbilical cords," to pass information between them. It was found that two cords are necessary, forming a mesh with a hole in it, as shown in Fig. 4. Again, finite-difference equations are used, where possible. At mesh boundary points, both interior and exterior, measured equations are written.

Since in this problem there are two conductors, metrons must be placed on both, and the integration in (3) taken over both conductors. The Green's function is simply that of a point charge over a ground plane (no dielectric in this problem). The principle is that the Green's function must satisfy boundary conditions that are external to the mesh, while the finite-difference formulation takes care of boundary conditions on conductors or dielectrics within the mesh.

There is more freedom in choosing metrons when there are two conductors. In general, the metron charge distributions are arbitrary. Thus, some metrons might include charge on only the first conductor, and others on the second. In fact, this works, provided that all points on the conductors are supplied with charge by at least one of the metrons. In this case, the metrons used were $\sigma_1 = 1$ on the top, $\sigma_2 = 1$ on the bottom, $\sigma_3 = \cos(2\pi x/W)$ on the top, $\sigma_4 = \cos(2\pi x/W)$ on the bottom and finally $\sigma_5 = \cos(4\pi x/W)$ on the top. The capacitances calculated using this method agree well with calculations using the spectral domain method [4].

VI. Conclusion

In this letter, we have demonstrated the use of the MEI technique for quasi-static field computation, in geometries typical of planar microwave circuits. We note that in this technique the computational mesh does not have to contain all conductors in the problem; information about conductors outside the mesh is supplied by Green's functions satisfying boundary conditions on those external conductors. Interestingly, the answers obtained depend little upon the choice of the metrons. Umbilical meshes, i.e, meshes with holes inside them, are convenient and efficient for problems having several conductors. For large problems, we expect that the rapidity with which the resulting sparse matrices can be inverted will far outweigh the initial time required in calculating the necessary Green's functions. Thus, overall solutions will be obtained more quickly than with earlier methods.

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